PD – 084

I Year M.Sc. (DCC) Degree Examination, January 2018
(Y2K13 Scheme) (Fresh and Repeaters)
MATHEMATICS
M101 : Algebra

Time : 3 Hours Max. Marks : 80

Instructions : *1) Answer* ***any 5*** *questions, choosing atleast* ***two*** *from* ***each*** *Part.*

2) ***All*** *questions carry* ***equal*** *marks.*

PART – A

1. a) Let $\phi : G \to \overline{G}$ be a homomorphism with Kernel K and let N be a normal subgroup of G. Then show that $\frac{G}{N} \approx \frac{\overline{G}}{\overline{N}}$. **5**

b) Prove that Inn (G) $\approx \frac{G}{Z(G)}$, where Inn (G) is a group of inner automorphisms of G and Z(G) in the centre of G. **5**

c) Show that every group in isomorphic to a subgroup of A(S), for some appropriate S. **6**

2. a) Verify the class equation for symmetric group S_3, by using generator-relation form. **5**

b) Prove that any two p-sylow subgroups are conjugate to each other. **6**

c) Let G be a group of order pq, where p and q are distinct primes with $p< q$ and $q \not\equiv 1 \pmod p$, then prove that G is abelian. **5**

3. a) Let R be a commutative ring with unity whose ideals are {0} and R only. Prove that R is a field. **5**

b) Let U be the left ideal of a ring R and $\lambda(U) = \{x \in R : x\,u = 0 \text{ for all } u \in U\}$. Prove that λ (U) is an ideal of R. **4**

c) Define a maximal ideal of a ring R. If R is a commutative ring with unity and M is an ideal of R, then show that M is a maximal ideal of R if and only if R/M is a field. **7**

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4. a) Define Euclidean ring. Prove that the ring Z[i] of Gaussian integers is an Euclidean ring. **5**

   b) State and prove unique factorization theorem. **5**

   c) State and prove Einstein criterion for irreducibility of a polynomial. **6**

PART – B

5. a) Let K be an extension of a field F and $a \in K$ be a algebraic over F and of degree n. Prove that $[F(a) : F] = n$. **6**

   b) Let $f(x) \in F[x]$ be degree $n \geq 1$. Then prove that there is an extension E of F of degree atmost n! in which f(x) has n-roots. **5**

   c) Define splitting field of a polynomial over a field F. Determine the splitting field of $x^3 - 2$ over the field Q. **5**

6. a) If F is a field of characteristic zero and a, b are algebraic over F, then prove that F(a, b) is a simple extension of F. **5**

   b) Define a perfect field. Show that any field of characteristic zero is perfect field. **6**

   c) If K is a finite Galois extension of a field F and if G (K, F) is a group of all F automorphisms of K_s then prove that $O(G(K, F)) = [K : F]$. **5**

7. a) Let V be finite-dimensional vector space over F, prove that $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not 0. **5**

   b) Define the range and rank of a linear transformation T. If V is finite dimensional vector space over F, then show that $T \in A(V)$ is regular if and only if T maps V onto V. **6**

   c) If V is n-dimensional vector space over F and if $T \in A$ (V) has all its characteristic roots in F, then show that T satisfies a polynomial of degree n over F. **5**

8. a) Prove that two nilpotent linear transformations are similar if and only if they have the same invariants. **5**

   b) Define a unitary transformation T. Prove that linear transformation, T on V is unitary if and only if it takes an orthonormal basis of V into an orthonormal basis of V. **4**

   c) State and prove Sylvester's law of inertia for real quadratic form. **7**

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